Jacob Palis

Professor at the Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro

2010 Balzan Prize for Mathematics (pure and applied)
For his fundamental contributions to the Mathematical Theory of Dynamical Systems.

Institution Administering Research Funds:
Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro

Adviser for the Balzan General Prize Committee: Étienne Ghys

Dynamical Systems, Chaotic Behaviour – Uncertainty, Linear Cocycles and Lyapunov Exponents

Jacob Palis is coordinating his Balzan Research Project together with Jean-Christophe Yoccoz at the Instituto de Matemática Pura e Aplicada, IMPA, Rio de Janeiro, Brazil. The creation of the modern theory of dynamical systems towards the end of the nineteenth century is attributed to Henri Poincaré. It is the principal mathematical approach used to model the evolution of many phenomena in nature. Classical examples are population growth of species, weather and climate prediction. Perhaps the same theory can be applied to understand certain aspects of turbulence in physics. Since Poincaré we have been wondering if it is possible to understand the typical behaviour of a typical dynamical system, where *typical* should be understood in a probabilistic sense to cover almost all possibilities.

Starting from a selected initial position of the system, one tries to describe the behaviour of its future trajectory, defined by its successive positions as time evolves. For example, the motion of the atmosphere is governed by a very complicated evolution equation, which cannot be solved explicitly. In 1963, Edward Norton Lorenz, a theoretical meteorologist, proposed a "toy" weather model, involving only three dimensions and intended to be much easier to understand. The question of knowing whether this oversimplified model still captures the main properties of the actual atmospheric motion is controversial among physicists and meteorologists. However, Lorenz was able to observe "chaotic behaviour" in his "toy" model. Minute changes in the initial

data used were shown to produce extremely radical changes in the outcome. This was very surprising at the time. Jacob Palis's research project proposes to tackle several conjectures which would imply that the phenomenon witnessed by Lorenz is not an exception but, on the contrary, may capture some fundamental features of general dynamics. The research project will study (and hopefully prove) a set of conjectures for dynamical systems that leads to a global perspective in this important branch of mathematics.

Towards the proof of this main conjecture concerning attractors of dynamical systems, in dimension one there was good progress on the finiteness of the number of attractors for typical dynamical systems in the case of maps of the interval. In fact, the conjecture is true even when there is a finite number of discontinuities, considering C3 maps having negative Schwarzian derivatives. The result is due to Brandão, Palis and Pinheiro. Relevant results have been achieved by Lyubich and Martens for diffeomorphisms in two dimensions in their work on the renormalization of Hénon attractors. In higher dimensions, for C1 diffeomorphisms of compact manifolds, the conjecture is essentially true by the result of Crovisier and Pujals. Progress in the differentiable case in two dimensions has also been achieved by Martens and Winckler for Lorenz Maps. Several other results are underway and shall be reported later. In brief, progress towards the proof of this hard conjecture is clear and steady.

An important contribution to the scientific popularization of dynamical systems, and particularly the main conjecture of the project concerning attractors, was made by Étienne Ghys in a fascinating film "Chaos" (http://www.chaos-math.org, http://www.chaos-math.org/it/chaos-i-moto-e-determinismo).

The Research Project is scheduled to take place over the period 2011-2015. Part of the funds of the project will support the activities of young researchers at IMPA in research on *Dynamical Systems, Chaotic Behaviour and Uncertainty*. Also, as part of the project, three Balzan Symposia were planned to take place, two of them at the Institut Henri Poincaré in Paris (2013 and 2015) and one at IMPA (2012). The first Palis-Balzan Symposium on Dynamical Systems was held at IMPA, Rio de Janeiro, in June 2012. These symposia are designed to review advances and to stimulate further progress along the lines of the research project.

Papers presented at the First Palis-Balzan Symposium on Dynamical Systems IMPA, 25-29 June 2012:

Artur Ávila - IMPA, Rio de Janeiro and CNRS, France - On the metric properties of Feigenbaum-Julia sets; Pierre Berger - CNRS, France - Zoology in the Hénon family from twin baby Hénon-like attractors; Christian Bonatti - Université de Bourgogne, Dijon - Foliated hyperbolicity; Sylvain Crovisier - CNRS, France - Newhouse phenomenon and uniformity of extremal bundles; Lorenzo Diaz - Pontifícia Universidade Católica do Rio de Janeiro (PUC) - Robust vanishing of all central Lyapunov exponents; Luiz Henrique de Figueiredo - IMPA, Rio de Janeiro - Images of Julia sets that you can trust; Nicolas Gourmelon - Université Bordeaux 1 - C' dichotomies between Newhouse phenomena and dominated splittings, at homoclinic points; Pablo Guarino - IMPA - Rigidity of Critical Circle Map; Alejandro Kocsard - Universidade Federal Fluminense (UFF), Niterói, RJ, Brazil - Distributionally uniquely ergodic diffeomorphisms; Andrés Koropecki - UFF, Niterói, RJ - Prime ends rotation number and periodic points; Yuri Lima - Weizmann Institute of Science, Rehovot, Israel - Stationary spaces of discrete groups: an Abramov formula; Jorge Eric López - IMPA - Stable projections of Cartesian products of regular Cantor sets; Michael Lyubich - SUNY at Stony Brook, USA - On homoclinic tangencies in the complex Henon family; Marco Martens - SUNY at Stony Brook, USA - On the hyperbolicity of Lorenz renormalization; Carlos Gustavo Moreira - IMPA, Rio de Janeiro - On the continuity of fractal dimensions of horseshoes in dimenson 3; Sheldon Newhouse - Michigan State University -The Lorenz equations: A survey of rigorous results; Maria José Pacífico - Universidade Federal do Rio de Janeiro - Fiber contracting maps versus Lorenz-like attractors; Vilton Pinheiro - Universidade Federal da Bahia, Brazil - Measures with historic behavior; Rafael Potrie - Universidade la República, Uruguay - Partial hyperbolicity and leaf conjugacy in nilmanifolds; Enrique Pujals - IMPA, Rio de Janeiro; Critical points for surfaces diffeomorphisms, abundance of periodic orbits and structural stability; Alvaro Rovella - Universidad de la República, Uruguay - Structural stability in dimension two; Martín Sambarino - Universidad de la República, Uruguay - Some questions, problems and remarks regarding C^r dynamics; Carlos Matheus Santos - CNRS, France - Fractal geometry of non-uniformly hyperbolic horseshoes; Waliston Luiz Silva - Universidade Federal de São João Del-Rei - On the geometry of horseshoes; Sebastian Van Strien - Imperial College London - On stochastic stability of expanding circle maps with neutral fixed points; J. Regis Varão - Universidade de São Paulo, São Carlos, Brazil - Center foliation: Absolute continuity, disintegration and rigidity; Marcelo Viana - IMPA, Rio de Janeiro - Time 1 maps of geodesic flows; Jiagang Yang - UFF Niterói, RJ - Diffeomorphisms with contracting Center.

The second Palis-Balzan Symposium on Dynamical Systems was held at the Institut Henri Poincaré, Paris, in June 2013.

Papers presented at the Second Palis-Balzan Symposium on Dynamical Systems, 10-14 June, 2013:
S. Alvarez - Institut de Mathématiques de Bourgogne, Dijon - Ergodic study of some foliations; A. Avila - IMPA, RJ and CNRS, France - The Billiard on the regular polygon; P. Berger - CNRS, France - Two results around the positive metric entropy conjecture for the standard map; C. Bonatti - Université de Bourgogne, Dijon - France - Pesin theory for C1-dynamics: a story of domination; P. Brandão - IMPA, Pós-doutorado - On the Finiteness of Attractors for Maps of the Interval Allowing Discontinuities; J. Buzzi - Department de Mathématiques d´Orsay, France - Diffeomorphisms without measures of maximal entropy; S. Crovisier - CNRS, France - Horseshoes with large entropy; V. Delecroix - l'Institut de Mathématiques, Paris VII. - Wind-tree models; W. de Melo - IMPA, RJ - Rigidity of Critical Circle maps; L. Diaz - Puc - Rio de Janeiro - Flip-flops in a blender. Robust existence of non-hyperbolic ergodic measures; N. Gourmelon - Université Bordeaux 1, France - Lyapunov exponents along periodic orbits for C^r generic dynamics; P. Guarino (USP) - Geometric rigidity of critical circle maps; A. Hammerlindl - University of Sidney - Ergodicity and Classification of Partially Hyperbolic Systems; A. Kocsard - UFF, Niterói, RJ - Livsic theorem for diffeomorphism cocycles; A. Koropecki - UFF,Niterói, RJ - Strictly toral dynamics; R. Leplaideur - Université de Brest, France - SRB measures for Almost Axiom-A diffeomorphisms. End of the story; Y. Lima - Weizmann Institute, Israel - Ergodicity of skew products over shifts of finite type; M. Lyubich - SUNY at Stony Brook, USA - Dynamics of dissipative polynomial automorphisms of C^2; M. Martens - SUNY at Stony Brook, USA - Generalized Henon Renormalization; C. Moreira - IMPA, RJ - Dynamical Markov and Lagrange Spectra and Geodesic Flows; S. Newhouse - Michigan State University, USA - A partial linearization theorem for three dimensional vector fields and applications; M. J. Pacífico - UFRJ, Rio de Janeiro - On measure expansive diffeomorphisms; R. Potrie - Centro de Matemática - Univ. de la Republica - Geometric properties of partially hyperbolic attractors; E. Pujals - IMPA, RJ - Minimality or density of periodic points for transitive attractors; J. Rodriguez-Hertz - Centro de Matematica - Universidade de La Republica, Uruguay - Partial hyperbolicity in dimension 3; M. Sambarino - Centro de Matematica - Universidade de La Republica, Uruguay - Dynamical coherence for partially hyperbolic diffeomorphisms istopic to Anosov; C. Matheus Santos - Collège de France, France - Finiteness of algebraically primitive closed SL (2,R)-orbits in moduli spaces; Y. Shi - Institut

de Mathématiques de Bourgogne-Dijon, France - Hyperbolic Dynamics on Heisenberg Nilmanifolds; S. Van Strien - University of Warwick, UK - Stochastic stability of expanding circle maps with neutral axed point; M. Viana - IMPA, RJ - Partially hyperbolic maps with 2-dimensional center; J. Yang - UFF, Niteroi, RJ - Dynamical behavior of ergodic measure along weak direction; A. Zorich - Université de Rennes 1, France - Right-angled billiards and volumes of moduli spaces in genus zero.

Researchers
Supervisor: Jean-Christophe Yoccoz, IMPA
A. Avila (Fields Medad 2014)
P. Berger
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Links:
First Palis-Balzan International Symposium on Dynamical Systems
http://www.impa.br/opencms/pt/eventos/store_old/evento_1203?link=2

Second Palis-Balzan International Symposium on Dynamical Systems
http://www.impa.br/opencms/pt/eventos/store_old/evento_1305?link=2